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FRACTURE MECHANICS AND THE TIME DEPENDENT

STRENGTH OF ADHESIVE JOINTS

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INTRODUCTION

As long as man has built composite structures, the joints connecting various parts have posed a limit to structural strength. This has been true of stitched, riveted or welded joints and is no less true of bondings with polymeric solids. Indeed composite materials depend to a very large extent on the strength of the bond connecting the various phases of the composite solid such as fiber reinforced plastics, asphalt concrete and solid propellant rocket fuel.

With regard to structural assembly of, say, aircraft components bonding offers advantages through possible advantages in manufacturing ease and weight saving, while the use of composite materials makes structural bonding almost mandatory because mechanical fasteners carry with them stress concentrations as sites of failure initiation. While the problem of bonding two fiber composite panels is directly dependent on the adhesion of the polymer matrix to the fibers and not only on the narrow region associated with the necessary "bond-line" joining the two panels, the following development will, nevertheless, add to understanding some salient features of that problem.

Specifically, we shall be concerned with the problem of time dependent failures of elastic or viscoelastic solids jointed by a polymeric bonding layer.

Like all other problems of strength, the bond strength has its origin in attractive forces between atoms or molecules. Much work has been done on this molecular aspect of bond strength as well as its relation to the surface tension of the bonding agent. Indeed so much work has been done in this regard that a few

1

references cannot do justice to the depth of understanding that has been developed from this viewpoint [1].

It is our aim in this work to steer away from molecular concepts and to treat the problem of adhesion on a size scale which is large compared to molecular dimensions. We may thus deal either with the microscopic scale on the order of surface roughness [2] or with the much larger scale of a complete joint [3].

From the standpoint of continuum mechanics the treatment of the problem at the microscopic or macroscopic level is not different. What is different, however, is the interpretation of at least one material property which we shall call, for lack of a better term at present, the fracture energy. Microscopically, we may view bond failure as originating from a small region which was not wetted by the bonding agent due to surface roughness. This region may be on the order of the surface roughness in size. The fracture may now occur into one of the weaker, joined materials, into the bonding solid or along an interface*. In either case we would conceive of the fracture energy as the work done in breaking inter-atomic bonds. This is a quantity which can therefore be calculated in principle from molecular considerations.

Macroscopically, i.e., on the size scale of a technical joint, we may have to consider the fracture energy as an average quantity determined by what happened microscopically near the small flaws at the interface. In other words the macroscopic fracture energy includes the little stress concentrations due to the

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* We shall consider the criterion as to where the fracture occurs later on.

microgeometry of the rough surface and the intrinsic properties of the joined materials at the microscopic level.

The choice of which viewpoint one wishes to take depends on the application one has in mind. If one is interested in studying how different material properties of the adherents and the bonding agents interact to form stronger bonds, one would probably be concerned primarily with the microscopic dimension. On the other hand, if one is in need of determining the effect of loading on a macroscopic joint such as is often the case in aircraft design, one would favor the macroscopic approach and determine the gross fracture energy in a suitably designed test. Comments in this regard are in order again after the details of the development have been presented.

As our immediate goal we shall apply the principles of fracture in viscoelastic materials to the failure of bond systems involving visco-elastic or elastic adherents and a viscoelastic bonding agent. Little attention has been given to joint failure from the viewpoint of fracture mechanics. Treatments presumably available in the literature [3, 4] emphasize the macroscopic aspects of technical joints without attention being given to the multiplicity of failure pheonomea that may occur. In particular, time and temperature dependence of the joint fracture process have not been discussed extensively. Our emphasis will therefore be on those aspects of the problem which add to our under-standing of joint failure, on the one hand, in relation to micro-scopic process, and on the other hand, in relation to the macroscopic stress fields acting on the joint system.

In most practical situations one is faced with one of two cases. On the one hand one may wish to join hard viscoelastic (elastic) solids by a hard, polymeric bonding material produce a high strength joint such as in aircraft construction or fiber composites. On the other hand one may need to join soft rubbery components to produce a highly compliant bond system to support relatively small loads. The bonding of solid propellant rocket fuel to the rocket casing is an example in point. In either case the full spectrum of viscoelastic relaxation times is not likely to be invoked as far as the analyses of stresses are concerned and one may therefore be able to deal with approximations in viscoelastic stress analysis. It should be pointed out, however, that such simplifications are a matter of convenience and not one of necessity or principle.

STRESS ANALYSIS OF IMPERFECT BONDS

Bonds, like homogeneous solids fail because invariably present flaws cause load variations of the stress field to induce crack growth. If one accepts the pre-existence of crack-like flaws the problem of failure determination is "reduced" to determining the condition(s) under which such a flaw will enlarge. In general it will also be necessary to establish the rate at which the flaw growth occurs. It is quite easy to demonstrate that when viscoelasticity is involved growth may occur so slowly that the joint will perform satisfactorily for the useful life of the total structure. For the purpose of joint life prediction it is therefore very important to establish the growth history of a flaw as a function of the applied loading sequence.

From the viewpoint of fracture mechanics the first step in understanding the flaw instability condition is to understand the stresses in the vicinity of such flaws. We have previously discussed [5] the variety of flaws that may exist in materials and how quanitative mathematical descriptions require one to suitably model such flaws. The same applies to flaws near or at interfaces. Let us assume for the sake of brevity and convenience that these flaws are sharp cornered and crack-like. Suppose further for the moment that the flaw is situated at an interface between two solids which possess distinctly different material properties (cf. Figure 1). Classical (visco)elasticity theory predicts that the stresses at the edge or tip of the flaw become infinitely large and oscillate from tension to compression with increasing frequency as the crack front is approached from within

5

either material [6,7]. Along with this alteration in stresses appears an alternation of the crack-opening displacement such that portions of the supposedly stress free crack surface interpenetrate. This theoretical behavior is physically not realistic and remains unexplained by classical elasticity. Muskhelishvili points out, however, that this result extends over such a small distance in the crack vicinity that it is of no practical consequence. Furthermore, the unreasonable behavior vanishes if the joined materials are incompressible or if they have the same properties. The latter case identifies the crack as being imbedded in a continuum.

It has been shown elsewhere [7,8] that the stresses ahead of the tip of a two-dimensional crack can be written as

$$\sigma_y = [2\pi x]^{-\frac{1}{2}} \left| K_1 \cos\left(\beta \ln \frac{x}{2c}\right) - K_2 \sin\left(\beta \ln \frac{x}{2c}\right) \right| + 0(1)$$

$$\tau_{xy} = [2\pi x]^{-\frac{1}{2}} \left| K_1 \sin\left(\beta \ln \frac{x}{2c}\right) + K_2 \cos\left(\beta \ln \frac{x}{2c}\right) \right| + 0(1) \qquad (1)$$

$$\beta = \frac{1}{2\pi} \ln \left[\frac{\mu_1 + \mu_2 \varkappa_1}{\mu_2 + \mu_1 \varkappa_2} \right]$$

where the notation is consistent with Figure 1. The subscripts refer to materials "one" and "two" respectively and

$$\mu = \text{shear modulus}$$

$$\varkappa = 3 - 4\nu$$

and the K_i are called the stress intensity factors.

6

For the single flaw at the interface between two infinite solids under crack-parallel shear q and under uniform tension p normal to the crack one finds, e.g., that [8]

$$K_1 = \sqrt{\frac{c}{4\pi}} \quad [p + 2\beta q]$$

$$K_2 = \sqrt{\frac{c}{4\pi}} \quad [2\beta p - q] \tag{2}$$

We note further that when $\nu_1 = \nu_2 = 1/2$ (incompressible solids) that $\beta = 0$ and therefore in this special case the stress (and displacement) oscillations vanish and one obtains, as for a crack in a homogeneous solid

$$\sigma_y = [2\pi x]^{-\frac{1}{2}} K_1$$

$$\tau_{xy} = [2\pi x]^{-\frac{1}{2}} K_2 \tag{3}$$

In our further discussions we shall ignore the anomalous behavior of stress oscillations or assume incompressible material response when applicable.

Having dispensed with an apparently mathematical quirk in stress analysis we turn now to a physical observation in joint fracture for which adequate mathematical analyses are not available, although the tools for such analysis exist. It is a common observation that if a cracked sheet is not subjected to loads normal to the crack the latter will not propagate along its original axis but at some angle γ (cf. Figure 2a). The same is true for cracks at an interface between two solids. For brittle or poorly ductile materials it is believed [9]

7

that cracks will extend along a line which is normal to the direction of the locally maximum tensile stress (cf. Figure 2). Since a like criterion should apply to cracks in the vicinity of bonds, we find that a crack situated, say, at the interface will under general loading tend to propagate away from the interface into one or the other of the joined solids. As to whether the crack can propagate under this condition depends on the strength characteristics* of the material into which the crack wants to propagate. If two joined materials have significantly different strength properties it is conceivable that if the bond line containing a small crack is subjected to a shear stress of, say, magnitude τ_0 the crack will not propagate. However, if the shear stress were reversed in direction while maintaining its magnitude τ_0 constant the crack would propagate into the other material (cf. Figure 3). This fact can also be easily demonstrated experimentally.

It should be borne in mind in this context that we are considering the propagation of a small crack in the vicinity of two different, relatively large solids. The consequences of the pre-ceeding observation with regard to technical joints involving three separate solids (cf. Figure 4) are three-fold. First, the argument as to whether the crack will tend to propagate into one or the other material still holds. Therefore, the type of applied load, tensile, shear or a combination, still determines the gross strength characteristics of a composite bond system. Second, the failure

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* We will identify these strength characteristics quantitatively later.

characteristics are different if bond failure occurs as the result of a single crack propagating or as the result of many flaws which grow until they interact to form a large one. For a single crack the change of loading conditions will have a more pronounced effect than for multiple flaws, the individual responses of which are somehow averaged out to produce the final, gross failure. Third, the change of the geometry -- switching from two solids of Figure 3 to the three solids in Figure 5 -- results in pronounced changes of the stress field at the tip of any crack in the bond vicinity. Let us illustrate this statement by one particular example, referring for a more detailed discussion to the literature [10, 11, 12]. Consider a single crack in the three-body and two-material composite in Figure 5, which contains a central crack in the bonding agent so that $c > h$. It can be shown rigorously [10, 11] that if the Young's modulus of the material "1" is much larger than that of the bonding agent "2" then the stresses at the crack tip are proportional to the applied stress σ_o and to the square root of the <u>bond half thickness h.</u> On the other hand, if the elastic properties of materials "1" and "2" are (nearly) the same, then the local crack tip stresses are still proportional to the stress σ_o but now proportional to the square root of the <u>crack half length c.</u> We see that the material properties have a pronounced effect on the stresses at the crack tip and consequently can have a strong effect on the failure behavior of bonded solids.

Let us summarize this qualitative description of the crack propagation at or near joints by observing that crack growth is controlled by the stresses at the tip of the crack. These stresses in turn are controlled by both the applied loads as well as the material

properties of <u>all</u> the surrounding materials. Furthermore the failure
of technical joints depends on whether failure is caused by a
progression of a single crack or by the interaction of many cracks
growing simultaneously. In short, there are many parameters which
control the failure of joints.

EQUATION GOVERNING TIME-DEPENDENT FRACTURE

The theory for fracture growth in (linearly) viscoelastic continua has been documented in references [5, 13-16]. It suffices for our present purposes to review the principles involved and to state the result.

When a crack propagates the high stresses at a point just ahead of the crack tip have to unload to zero as the crack tip passes that point -- provided the crack surfaces are stress free and not pressurized. This unloading process is the result of the material disintegration at the crack tip through void formation [5]; the latter process may vary with regard to size scale, sometimes visible with the unaided eye, and sometimes only under high magnification. The net result of this process is that the unloading forces at the crack tip do work while acting through the displacements of the newly created crack surfaces. If the region ahead of the crack over which material disintegration takes place is very small, then it can be shown [13, 15] that the work done by the unloading stresses at the crack tip is equal to the work required to break the molecular bonds ahead of the crack. Because the stresses and displacements at the tip of a crack are, in general, time dependent and also a function of the crack tip velocity, the relation of time dependent loading and resultant time dependent crack propagation are implicitly accounted for.

In reference 16 we have derived the general equation for crack growth. For the limited case that loads on a fracturing

structure do not change in a sudden fashion* the mathematical

equation describing the physical fracture process just outlined is

$$D(\frac{\alpha}{\dot{c}}) \quad K^2 [c(t), t] = \Gamma \tag{4}$$

where

$D(t)$ = creep compliance of the homogeneous solid

α = small length over which material disintegration takes place, a microstructural parameter; see below

$c, \dot{c}$ = crack size and crack tip velocity, both functions of time

Γ = intrinsic, constant surface energy

$K[c, t]$ = crack tip stress intensity factor

Before proceeding to the application of viscoelastic fracture theory to

the failure of joints a comment is in order with respect to the effect

of the detailed stress distribution at the tip of the crack. We recall

that a fundamental ingredient to equation (5) is the work done by the

forces at the crack tip during the unloading process. This process

occurs over the small distance α. The total work is thus an integral

over the distance α ahead of the crack and it is the integral of the

stress and displacement distribution which enters equation (1).

Inasmuch as the integral smooths out details of the stress and dis-

placement distribution an approximation may be quite acceptable--

as long as the approximate stress and the displacement fields are

compatible in the sense of viscoelastic stress analysis.

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$$\frac{\alpha}{\dot{c}} \frac{d\sigma}{dt} << 1; \quad \text{(see reference 16)}$$

12

EQUATION GOVERNING TIME-DEPENDENT JOINT FRACTURE

We shall not here derive the equation equivalent to (5) for joint fracture but deduce it from existing results. For this purpose it is convenient to assume, for the present, that fracture occurs along an interface. We note further that for an elastic solid instead of a viscoelastic one equation (4) reduces to

$$\frac{1}{E} \, K^2(c) = \Gamma \tag{5}$$

where the elastic compliance $1/E$ has replaced the creep compliance $D(\frac{a}{c})$ in accordance with Griffith's result. Alternately, we can look upon the viscoelastic equation (4) as a generalization of the elastic result (5).

Mossakovskii and Rybka [11] have given the instability criterion for two elastic solids unbonded in a circular region and under tension normal to the interface. For the special case of incompressible elastic solids their result becomes particularly simple, namely,

$$\frac{1}{2} \left[\frac{1}{E_1} + \frac{1}{E_2} \right] K^2 [c, E_1 \, E_2] = \Gamma \tag{6}$$

where E_1 and E_2 are the Young's moduli for the two materials joined and Γ is the energy required to form a unit of new surface. We notice that the major difference in the fracture and the joint problem is that the average elastic compliance of the joint problem has replaced the single compliance of the fracture equation (5).

We deduce that equation (6) can be generalized for the viscoelastic case as (5) can be generalized into (4). We find therefore, that if two poorly compressible viscoelastic solids are joined the rate of unbonding $\dot{c}$ is given by

$$\frac{1}{2}\left[D_1(\frac{\alpha}{\dot{c}}) + D_2(\frac{\alpha}{\dot{c}})\right]K^2\,[c,1,2\,] = \Gamma \tag{7}$$

where D_1 and D_2 are the creep compliance of the two joined solids. The fact that the stress intensity factor K depends, in general, on the material properties of the two solids is expressed by the numerals 1 and 2 in the argument of K. This generalization is also summarized in Figure 6. Note that the time-dependent description (7) of the joint fracture specializes to the time-dependent fracture of a homogeneous continuum and to the elastic cases.

ILLUSTRATION OF TIME-DEPENDENT EFFECTS AND TRANSITION FROM ADHESIVE TO COHESIVE FRACTURE

Consider two viscoelastic solids bonded together imperfectly; let their time-dependent behavior be characterized by the creep compliances as shown in Figure 7. Assuming, again for reasons of simplicity in presentation the applied loading to be such that unbonding occurs along the interface*, we can use equation (7) to calculate the rate of unbonding $\dot{c}$ as a function of the parameter $K/\sqrt{\Gamma}$. This relation is shown in Fig. 8. Since the stress intensity factor K is proportional to the magnitude of the applied load we see an increase in load will bring about a much more than proportional increase in rate of unbonding. This fact may be illustrated also in a more conventional way by asking how much time Δt is required for the unbond to grow by a small, but detectible amount Δc. If the growth is so small that the stress intensity factor is not markedly affected, then we need not integrate the differential equation (7) but use the approximate relation

$$\Delta t \doteq \frac{\Delta c}{\dot{c}}$$

The result of such a calculation as derived from Figure 8 is shown in Figure 9 as the solid line, which is, upon noticing the proportionality between load and stress intensity factor K, a plot of how the applied load controls the failure time.

Two general observations should be made at this point. First we note that the abscissa contains the fracture energy Γ. This fracture energy which can be related to the molecular bonding process is a direct measure of the joint strength. Assume we wish

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*We shall subsequently deal with cases that deviate from this condition.

failure to occur at (or after) some particular time t_f (cf. Figure 9). If we increase Γ, then K, and therefore the applied load, must be increased to achieve failure in the same time. Alternately, merely increasing Γ will increase the failure times Δt for comparative loads (K). Note, that a relatively small increase in Γ will cause a large change in the failure time. To speak of a stronger joint we must thus be aware that "stronger" can mean either a higher load at comparative timescale or longer failure times at comparative stress levels.

Second, we note that the timescale on Figure 9, which is typical for polymeric solids, is so large that only a portion of the whole abscissa would be observed in a laboratory. However, it is well known that polymers respond to temperature changes with a change in response time [18]*. It follows from the thermorehological behavior, whether this behavior be simple or not, that an increase in temperature shifts the failure time curve to the left, leading to apparent weakening, while the opposite is true for temperature lowering. This fact has been observed experimentally by Wegman and Tanner [19].

Finally we must remark that we have considered in Figure 9 only the simplest explicit time-dependence of the jointed failure inasmuch as we have calculated only the time to propagate the crack some small (unspecified) distance. Joint failure is the result of

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* It should be remembered that temperature variations set up thermal stresses in joined materials having different thermal expansion properties. These thermal stresses will produce a contribution to the stress intensity factors which in turn will affect the failure time. Although thermal stresses are very important for determining joint integrity we wish to speak here only of the thermorheological material properties.

substantial crack growth which can be calculated by integrating the
non-linear differential equation (7) after the stress intensity
factor K is known as a function of the current flaw size c(t). The
latter determination is important because it combines the effect of
loading and geometry to determine whether the crack accelerates
[5, 15] or propagates at a steady, possibly slow speed [13].

INFLUENCE OF THE FRACTURE PATH ON JOINT STRENGTH

Having illuminated exclusively the time-dependence of the joint
failure process for the speical case of interface separation we should
free ourselves from that restriction and consider the consequences in
terms of the unbonding equation (7). Let us denote the properties
of the two joined solids by the numerals "1" and "2" and the property
of the interface by "3".

Basically two conditions influence the propagation of a crack.
First, the magnitudes of the fracture energy of the material through
which the crack propagates is significant. Second, often combined
sets of forces act on the crack tip such that one set tends to open the
crack and the other tends to cause shear along the crack surfaces. If
such a combination is just sufficient to cause crack growth -- or to
cause crack growth at some rate $\dot{c}$ -- then anyone of the two sets
of forces by themselves will not be sufficient to cause failure -- or
it will cause growth at a lower rate. This fact is implicit in the
results found by Erdogan and Sih [9]. We must therefore
consider the path or direction of fracture in addition to its rate
of propagation. Consider Figure 10a. If we assume that the
separation occurs at the interface* then equation (7) applies directly
as we had assumed earlier, except that now we should add a subscript
3 on to Γ. We have then

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*From a practical viewpoint this is undesirable because it indicates
that full advantage of the strength properties of the adherents has
not been exploited.

$$\frac{1}{2}\left[D_1\left(\frac{\alpha}{\dot{c}}\right) + D_2\left(\frac{\alpha}{\dot{c}}\right)\right]K^2 = \Gamma_3 \tag{8}$$

Now suppose that $\Gamma_3 > \Gamma_1$ and further that the stress intensity factor does not change if the crack is situated an infinitesimal amount ε away from the interface into material 1 (cf. Figure 10b). Equation (7) reads now

$$\frac{1}{2}\left[D_1\left(\frac{\alpha}{\dot{c}}\right) + D_2\left(\frac{\dot{c}}{\dot{c}}\right)\right]K^2 = \Gamma_1 \tag{9}$$

It is easy to show that under the condition $\Gamma_3 > \Gamma_1$ the velocity of crack growth $\dot{c}$ as calculated from equation (9) will exceed that calculated from equation (8). It would seem reasonable therefore that the crack will follow the path in material 1.

Next let us consider what happens if the loading is such that the crack propagates towards the bond line (Figure 10c) and let $\Gamma_2 > \Gamma_3 > \Gamma_1$. We first observe that to bring about the change to go from the case of Figure 10b to that of Figure 10c we had to add some load in accordance with the second of the conditions discussed at the beginning of this section. Second, upon meeting the interface the crack may not possess a stress intensity factor high enough to satisfy the equation

$$\frac{1}{2}\left[D_1\left(\frac{\alpha}{\dot{c}}\right) + \frac{1}{2}\left(\frac{\alpha}{\dot{c}}\right)\right]K^2 = \Gamma_2 \tag{10}$$

but may be high enough to satisfy the equation

$$\frac{1}{2}\left[D_1\left(\frac{\alpha}{\dot{c}}\right) + D_2\left(\frac{\alpha}{\dot{c}}\right)\right]K^2 = \Gamma_3. \tag{11}$$

Accordingly the crack will propagate along the interface but not into the adherent 2. Of course a condition can be found on the load such that the crack will not propagate at all but be stopped by the second adherent.

Finally, let us consider more explicitly the effect of rate of unbonding upon the transition between adhesive and cohesive failure of a simple structural joint. Figure 11 shows such an idealized arrangement of a viscoelastic solid bonded to a rigid substate by a viscoelastic bonding agent. Since we have already treated the condition which determines whether the fracture will propagate in the adhesive or along the interface we will now have to consider only the condition under which failure occurs at or near the interfaces 3 or 4.

If unbonding occurs at the interface 3 then the governing equation is

$$\frac{1}{2} \left[D_1(\frac{\alpha}{\dot{c}}) + D_2(\frac{\alpha}{\dot{c}}) \right] K^2 [1:2; c(t)] = \Gamma_3 \tag{13}$$

while the corresponding equation for unbonding the interface 4 is

$$\frac{1}{2} D_2(\frac{\alpha}{\dot{c}}) K^2 [1; 2; c(t)] = \Gamma_4 \tag{14}$$

We presume for the present that the small structural parameter c is the same for both interfaces, a condition which can later be relaxed at the expense of introducing another variable into the problem.

With the restrictive understanding that flaws at either interface are to give rise to approximately equal stress intensity factors we may now consider these cases as illustrated in Figure 12. There we have plotted the rate of bond destruction for three different sets of adhesive fracture energy values the dotted curves corresponding to unbonding of the rigid solid while the solid curve describes the separation of the two viscoelastic solids along interface 3. It is

20

clear from Figure 12a and 12c that preferential unbonding will occur under all loading conditions at one or the other interface if the separation energies are distincity different. However, if the separation energies are nearly equal (cf. Figure 12b) then unbonding can occur along either interface depending on how high the stress intensity factor is. In other words, unbonding can occur on either interface depending on how hard one pulls on the assembly. This deduction may also be the reason for the observed phenomenon [20] that certain systems peel cleanly at a given rate (given force) while a complex or intrabond failure is observed for higher peel rates (higher peel forces).

We may now relax the condition that the structural size parameter be the same for both interfaces and demonstrate as an example the effect of a larger parameter for the rigid-polymer interface 4. This is illustrated qualitatively in Figures 12 d-f, again for the three relative values of the separation energies Γ_3 and Γ_4. We see in Figure 12d that although $\Gamma_3 > \Gamma_4$, the failure may change from unbonding at the interface 3 to debonding the interface 4 depending on the stress level.

CONCLUDING REMARKS

We have attempted to elucidate the time or rate dependent fracture of adhesive joints from the viewpoint of viscoelastic fracture mechanics. No phenomena, not hitherto observed experimentally, are reported. No further assurance is given that the problem of bonding is a technical trivial problem and the chemistry of surface preparation has been avoided.

However, we have attempted to elucidate, on the basis of a few principles of fracture mechanics, a variety of observed phenomena. Thus the hope exists that the large variety of apparent physical phenomena can be reduced by viewing them from the umbrella viewpoint of fracture mechanics.

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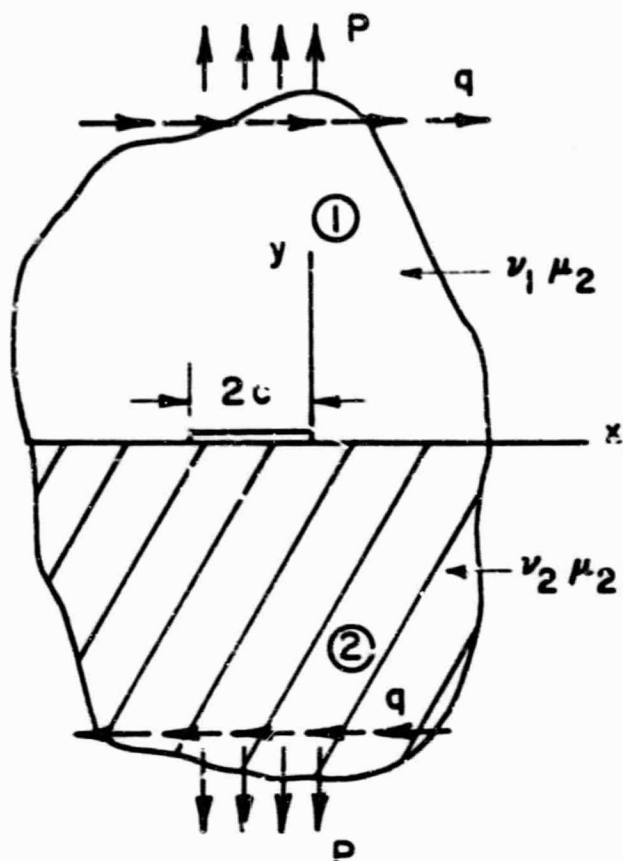
FIG. I CRACK AT A MATERIAL INTERFACE
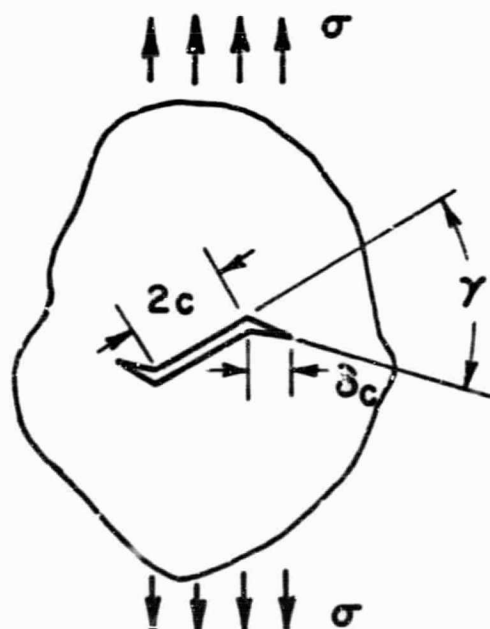μ = SHEAR MODULUS
ν = POISSON'S RATIO

FIG. 2 ANGULAR CRACK PROPAGATION UNDER
COMBINED LOADING

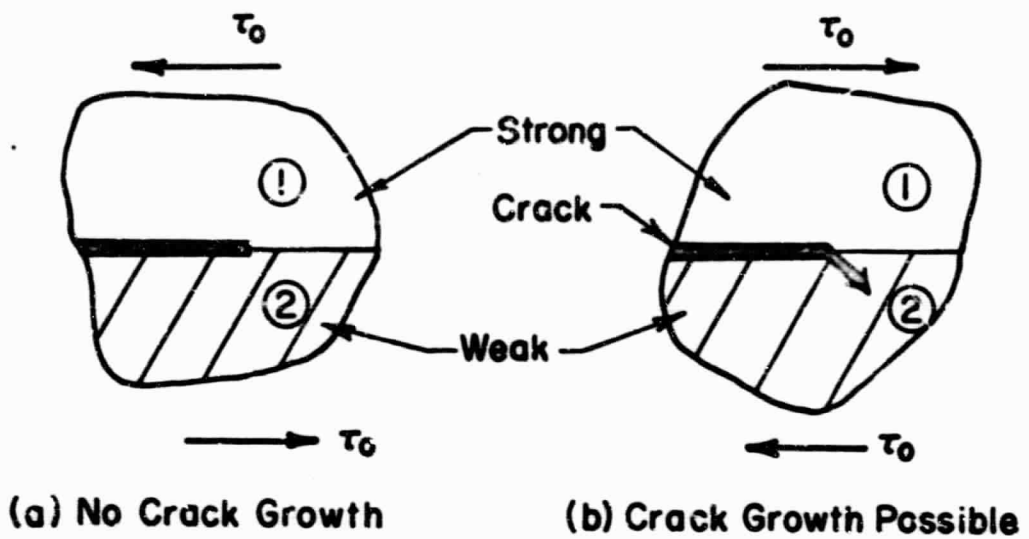(a) No Crack Growth   (b) Crack Growth Possible

FIG. 3 EFFECT OF SHEAR STRESS ORIENTATION
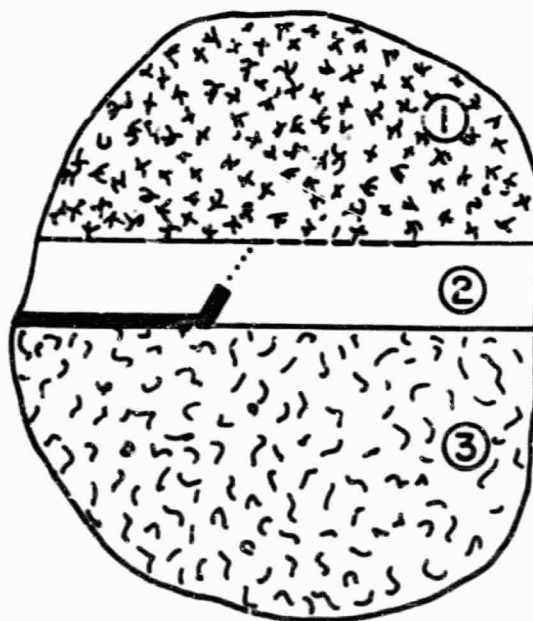ON CRACK PROPAGATION

FIG. 4 CRACK PROPAGATION IN A TECHNICAL
JOINT

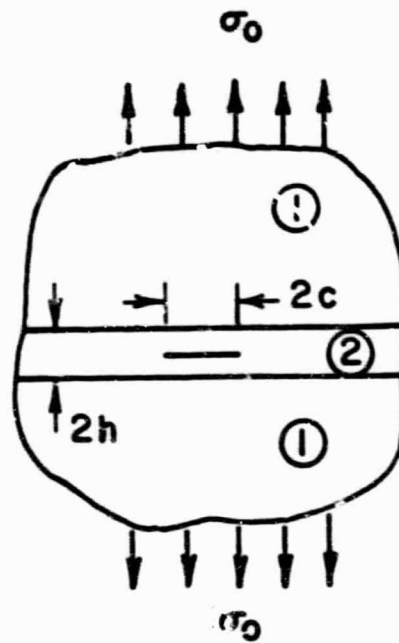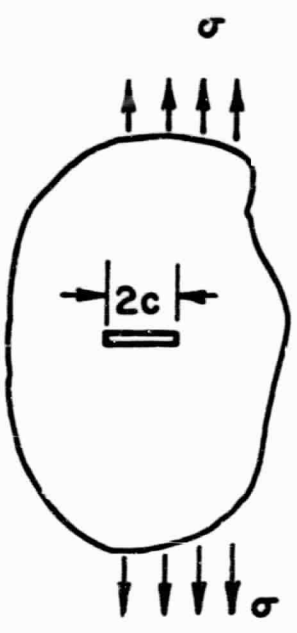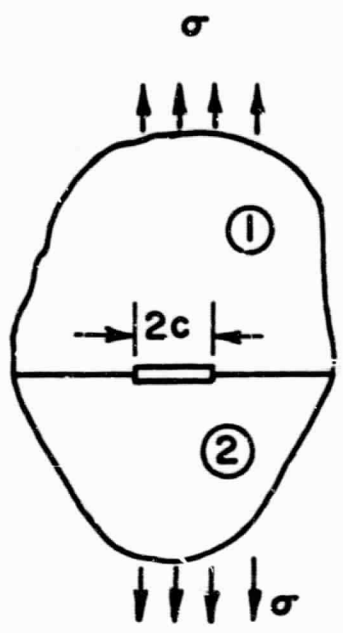FIG. 5 COMPOSITE BOND SYSTEM CONTAINING
A SMALL FLAW

FIG. 6 FRACTURE AND UNBONDING EQUATIONS FOR
HOMOGENOUS AND INHOMEGENOUS SOLIDS
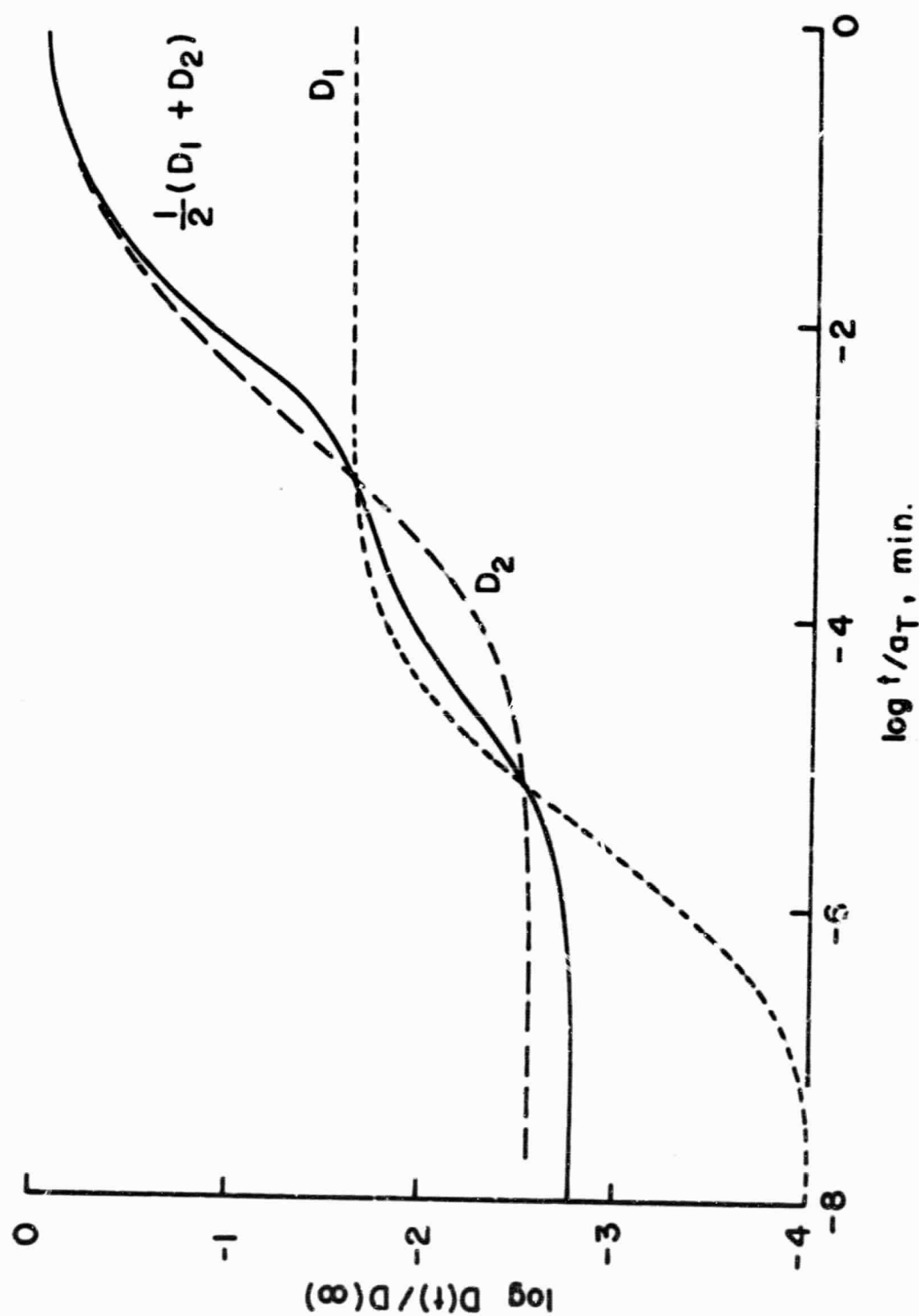
FIG. 7 TWO TYPICAL CREEP COMPLIANCES AND THEIR AVERAGE

62
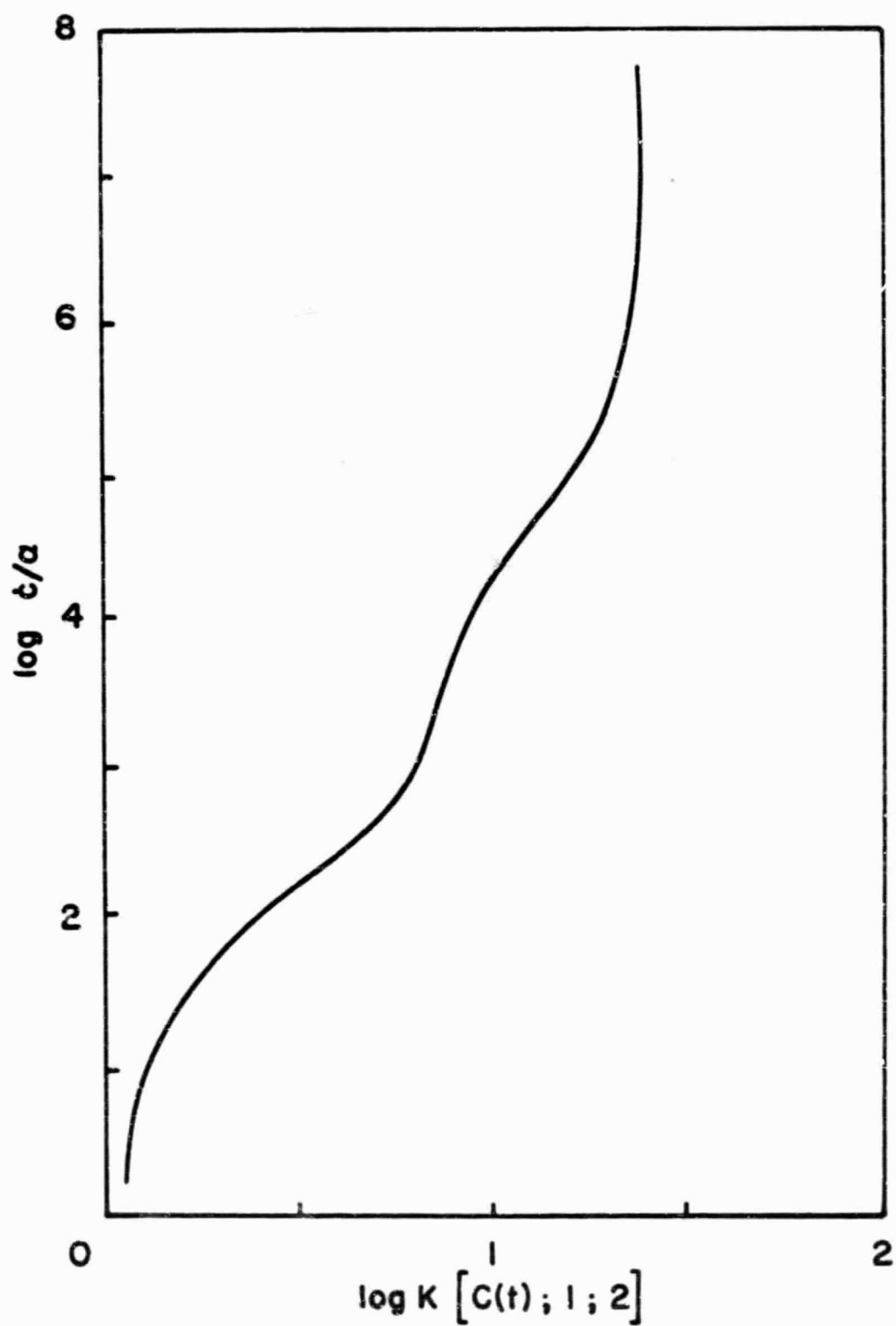
FIG. 8 UNBONDING RATE AS A FUNCTION OF
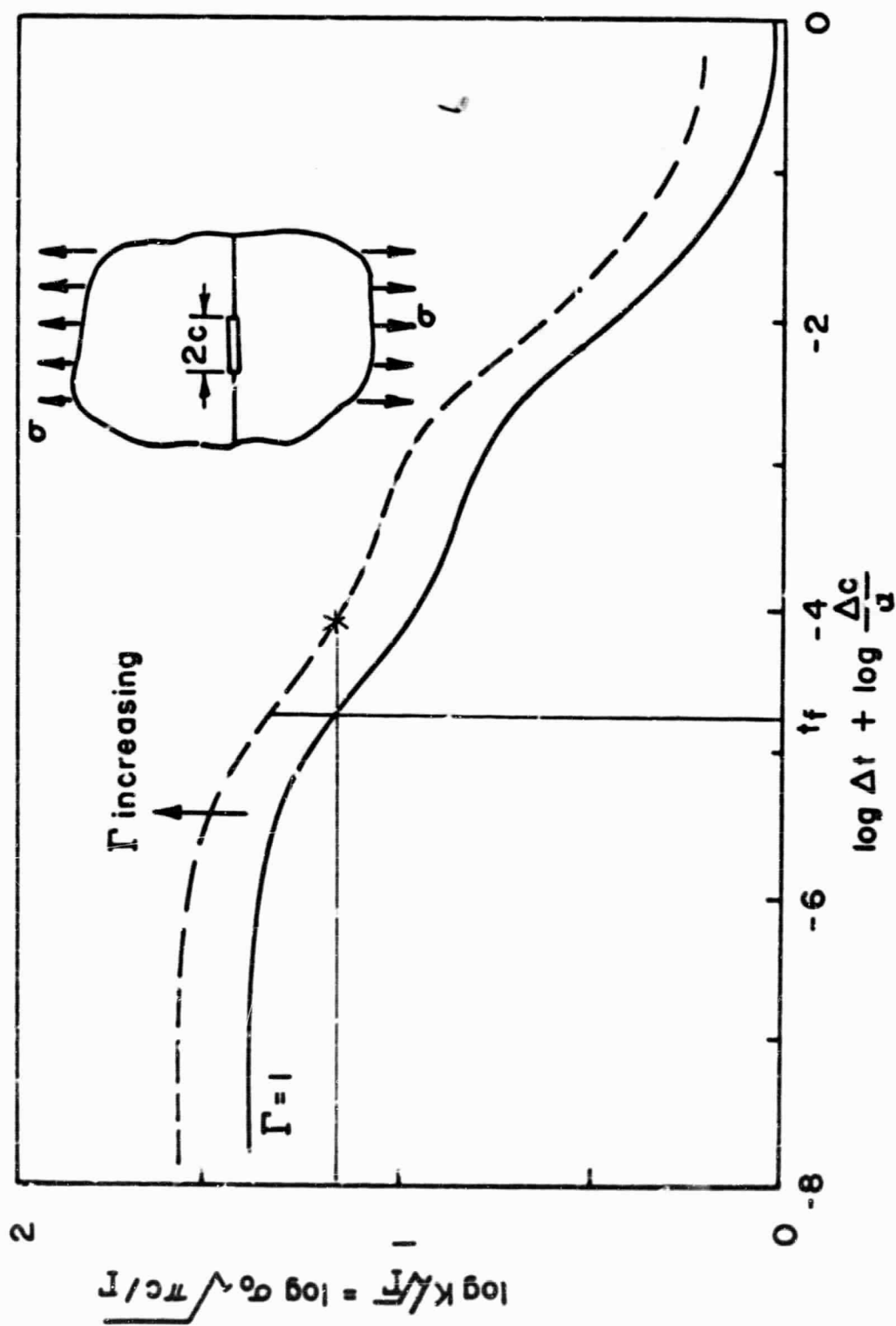THE STRESS INTENSITY FACTOR

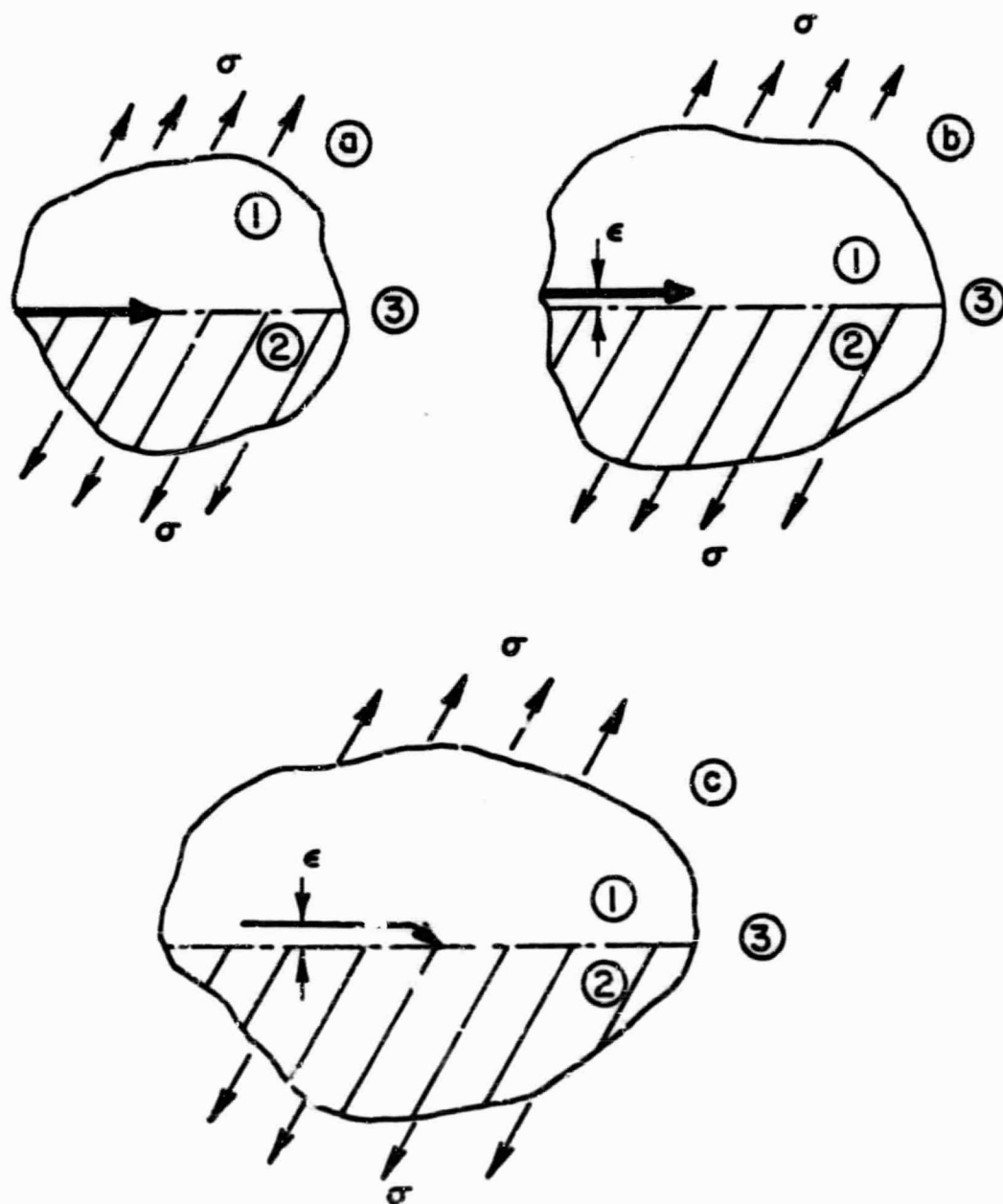FIG. 9 FAILURE TIME OF A JOINT (Δt = TIME TO OBSERVE NOTICEABLE CRACK GROWTH Δc)

$$\log K\sqrt{\Gamma} = \log \sigma_o \sqrt{\pi c/\Gamma}$$

$$\log \Delta t + \log \frac{\Delta c}{a}$$

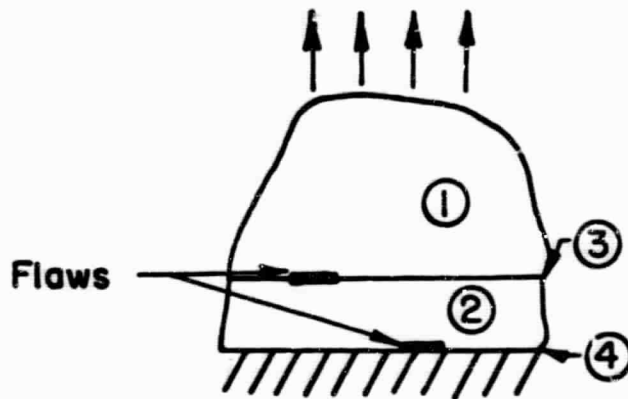FIG. 10 FRACTURE PATHS IN THE VICINITY
OF AN INTERFACE

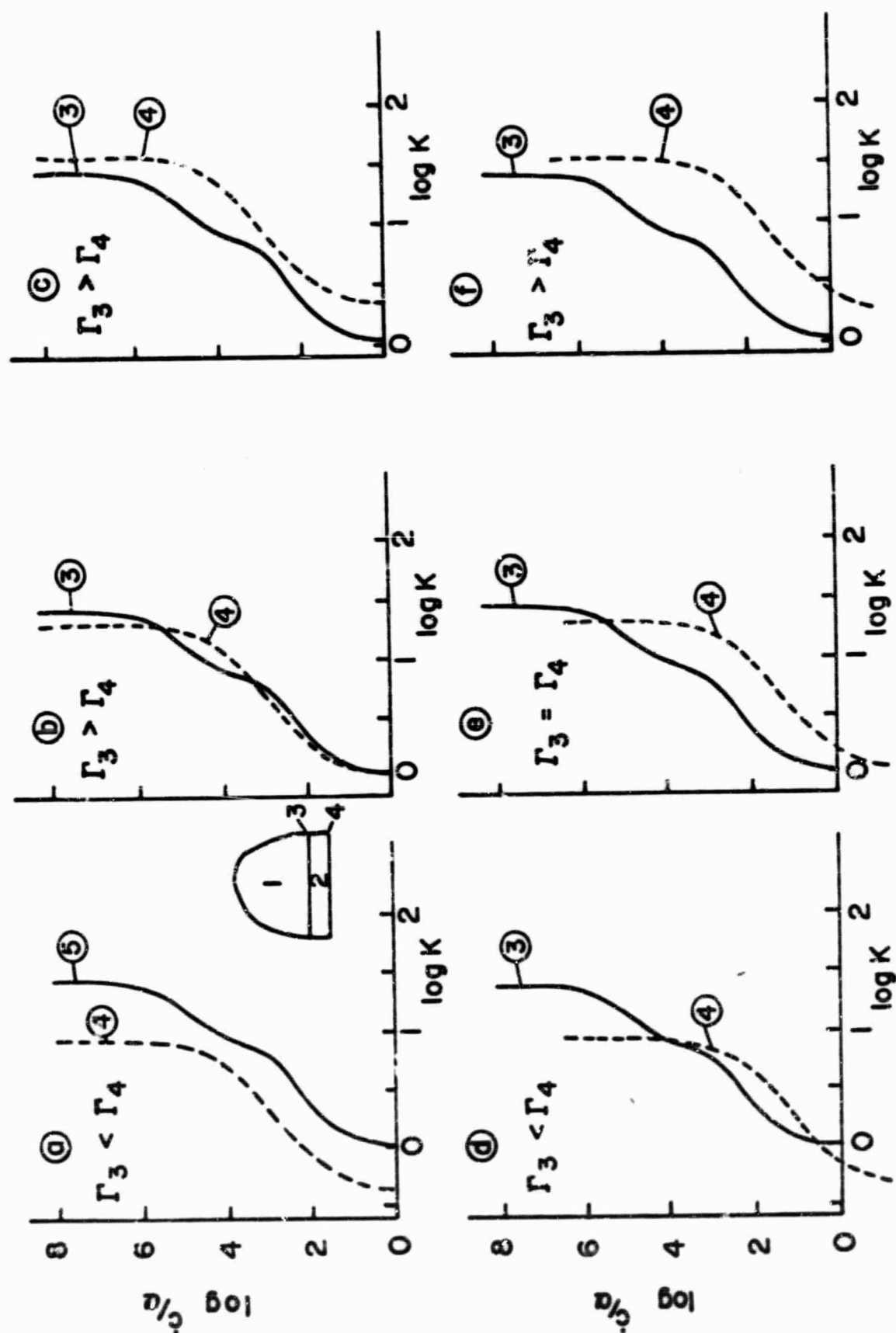FIG. II GEOMETRY OF A SIMPLE STRUCTURAL
JOINT

FIG. 12 COMPETITIVE RATES OF UNBONDING ċ OF TWO INTERFACES IN DEPENDENCE ON SURFACE ENERGY AND THE STRUCTURAL PARAMETER α

34